The Role of Mathematics in Physics

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GENETIC v. AXIOMATIC METHOD

Natural no's
→ integers → Rationals
↳ reals

Also Complex no's
geometry etc

These provide 'Concrete'
realizations or representations
of Abstract Structures

INTENSIONAL V. EXTENSIONAL

AXIOMATICS

Categorical

Ex Peano Arithmetic

Categorical

Vector Space
Hilbert Space
(of given dimension)

Non-Categorical

Group
Field
Banach Space

↓

May have distinct Concrete realizations

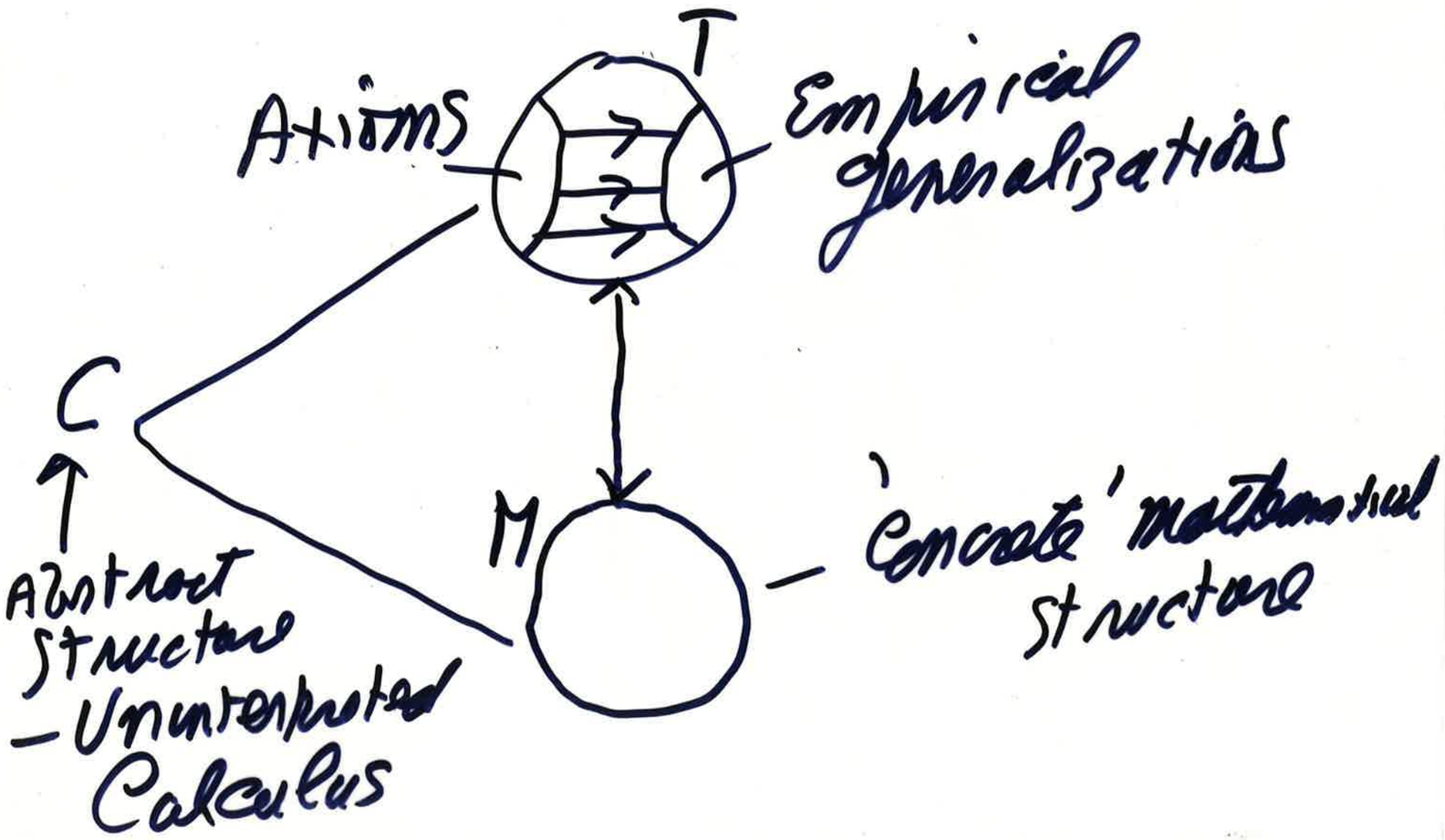
Ex L_2 - space → Wave Mechanics
l_2 - space → Matrix Mechanics

Mathematical Models in physics are concrete realizations of categorical Abstract Structures

What distinguishes axiomatized Mathematical structures from arbitrary axiomatized structures?

Ans (?) (Concrete realization in terms of Mathematical objects — constructed ultimately from Numbers

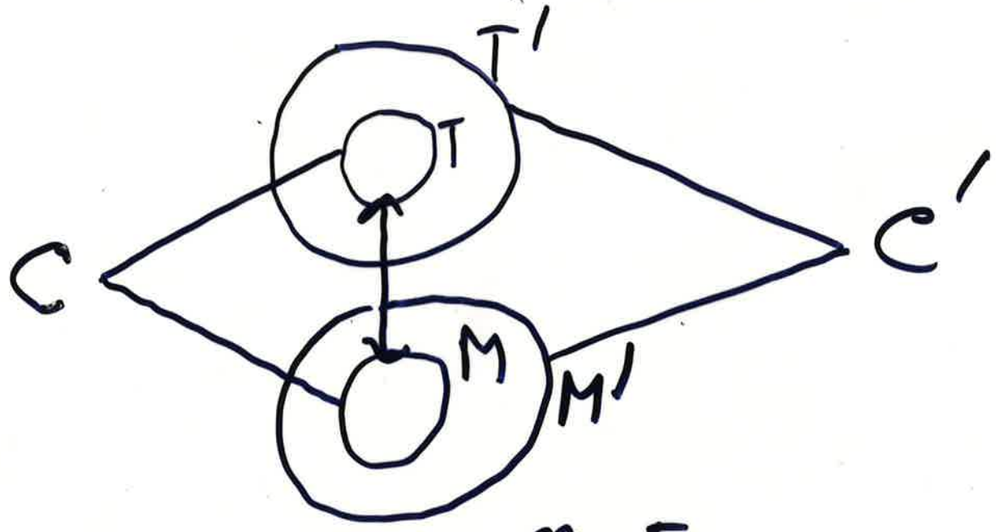
Relation of Mathematics to Physics



USE OF NON-CATEGORICAL STRUCTURES

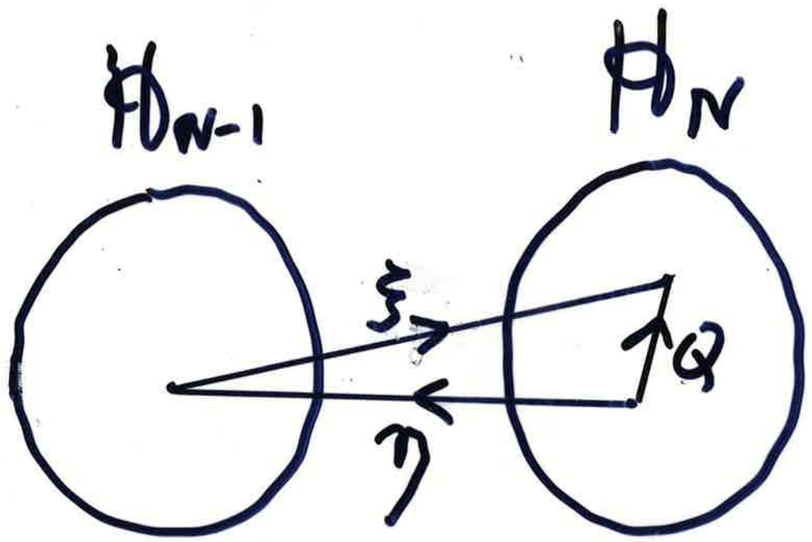
Eg Groups - Economy of not repeating same argument in many different contexts

Different ways of formulating a theory in terms of <u>Surplus Structure</u>



Eg's Analytic S-Matrix
Second Quantization

FOCK SPACE



write $Q = \xi\eta$

Heuristic Role of Surplus Structure

Eqs Quantum Field Theory
Hole Theory & positrons
Gauge theories
S-Matrix theory

Why is Mathematics Physics Successful?

Deals with quantitative aspects of the world.
But what about Hilbert space and Riemannian geometry?

Also Problems amenable to Mathematics first to be treated — classical celestial mechanics v. Nuclear Physics

The Computation Gap

'Empirical' mathematics
– approximations justified
in terms of successful
predictions
Ex Quantum Chemistry?

The Role of the Computer

Allows more sophisticated approximations and theoretical models to be explored.

Rigour in Mathematics

Gauss

$$\iint (lX + mY + nZ)\, dS = \iiint \left(\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z}\right) dV$$

Stokes

$$\int \left(X \frac{dx}{ds} + Y \frac{dy}{ds} + Z \frac{dz}{ds}\right) ds$$

$$= \iint \left\{ l\left(\frac{\partial Z}{\partial y} - \frac{\partial Y}{\partial z}\right) + m(\quad) + n(\quad) \right\} dS$$

Modern version

$$\int_{\partial \Phi} \omega = \int_{\Phi} d\omega$$

STERILITY IN THEORETICAL PHYSICS OF TOO MUCH RIGOUR

See Dirac S-function

But balance against sloppy or incoherent reasoning

The Nature of Idealization

Addition of ideal elements
- destruction from Abstraction
- cf. notion of surplus structure as above.

Modern Mathematics
Golden Age or
Age of Decadence ?

Roots of significant mathematics in 'concrete' realizations

INTERACTION BETWEEN MATHS AND PHYSICS

Conic sections — Kepler
Hilbert Space — QM
Riemannian geometry — G.R.
etc.

but also
Development of Calculus
Fourier analysis
etc

Successes of Mathematical Physics

Ground state of He

Lamb shift

Anomalous magnetic moment of electron

↳ Expt: $(115\,965\,24 \pm 2) \times 10^{-10}$

Theory: $(115\,965\,24 \pm 6) \times 10^{-10}$

How is this possible?